

## Probability of a Binomial Event

The BINOMIAL program is a short program to calculate the probability of obtaining k or fewer occurrences of a dichotomous variable out of a total of n observations when the probability of an occurrence is known. For example, assume a test consists of 5 multiple choice items with each item scored correct or incorrect. Also assume that there are five equally plausible choices for a student with no knowledge concerning any item. In this case, the probability of a student guessing the correct answer to a single item is 1/5 or .20. We may use the binomial program to obtain the probabilities that a student guessing on each item of the test gets a score of 0, 1, 2, 3, 4, or 5 items correct by chance alone.

The formula for the probability of a dichotomous event k where the probability of a single event is p (and the probability of a non-event is q = 1 - p is given as:

$$P(k) = \frac{N!}{(N - k)! k!} p^{(N-k)} q^k$$

For example, if a "fair" coin is tossed three times with the probabilities of heads is p = .5 (and q = .5) then the probability of observing 2 heads is

$$\begin{aligned} P(2) &= \frac{3!}{(3-2)! 2!} 0.5^1 \times 0.5^2 \\ &= \frac{3 \times 2 \times 1}{1 \times (2 \times 1)} \times 0.5 \times 0.25 \\ &= \frac{6}{2} \times 0.125 = .375 \end{aligned}$$

Similarly, the probability of getting one toss turn up heads is

$$P(1) = \frac{3!}{(3-1)! 1!} 0.5^2 \times 0.5 = \frac{6}{2} \times 0.25 \times 0.5 = .375$$

and the probability of getting zero heads turn up in three tosses is

$$P(0) = \frac{3!}{(3-0)! 0!} 0.5^0 \times 0.5^3 = \frac{6}{6} \times 1.0 \times 0.125 = 0.125$$

The probability of getting 2 or fewer heads in three tosses is the sum of the three probabilities, that is, 0.375 + 0.375 + 0.125 = 0.875.

Shown below is the form used to obtain binomial probabilities and an example run of the procedure:

Binomial Probability of A events out of B

Enter the frequency observed in category A: 3

Enter the frequency observed in category B: 7

Enter the theoretical proportion expected in category A: 0.5

Plot the distribution (if A+B < 35)

Clear

Cancel OK

**Figure 1. Binomial Probability Form**

Binomial Probability Test

Frequency of 3.00 out of 10.00 observed

The theoretical proportion expected in category A was 0.500

The test is for the probability of a value in category A as small or smaller

than that observed given the expected proportion.

Probability of 0 = 0.0010

Probability of 1 = 0.0098

Probability of 2 = 0.0439

Probability of 3 = 0.1172

Binomial Probability of 3.00 or less out of 10.00 = 0.1719

Probability of more than 3.00 out of 10.00 = 0.8281