### **Proportion Differences**

A most common research question arises when an investigator has obtained two sample proportions. One asks whether or not the two sample proportions are really different considering that they are based on observations drawn randomly from a population. For example, a school nurse observes during the flu season that 13 eighth grade students are absent due to flu symptoms while only 8 of the ninth grade students are absent. The class sizes of the two grades are 110 and 121 respectively. The nurse decides to test the hypothesis that the two proportions (.118 and .066) do not differ significantly using the OpenStat program. The first step is to start the Proportion Differences procedure by clicking on the Statistics menu, moving the mouse to the Comparisons option and the clicking on the Proportion Differences option. The specification form for the test then appears. We will enter the required values directly on the form and assume the samples are independent random samples from a population of eighth and ninth grade students.

| Test of the Equality of Two Proportions  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|
| Data Entry By:       Assume:            • Values Entered On This Form         • Independent Proportions         • Dependent Proportions         • Depe |  |  |  |  |  |  |
| Sample 1 Freq.: 13 Sample Size: 110  |  |  |  |  |  |  |
| Sample 2 Freq.: 8 Sample Size: 121   |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Percent Confidence Interval: 95 Reset Cancel Continue  |  |  |  |  |  |  |

Figure 1 Testing Equality of Two Proportions

When the nurse clicks the Continue button the following results are shown in the Output form:

COMPARISON OF TWO PROPORTIONS

Test for Difference Between Two Independent Proportions

Entered Values

```
Sample 1: Frequency = 13 for 110 cases.
Sample 2: Frequency = 8 for 121 cases.
Proportion 1 = 0.118, Proportion 2 = 0.066, Difference = 0.052
Standard Error of Difference = 0.038
Confidence Level selected = 95.0
z test statistic = 1.375 with probability = 0.0846
z value for confidence interval = 1.960
Confidence Interval: ( -0.022, 0.126)
```

The nurse notices that the value of zero is within the 95% confidence interval as therefore accepts the null hypothesis that the two proportions are not different than that expected due to random sampling variability. What would the nurse conclude had the 80.0% confidence level been chosen?

If the nurse had created a data file with the above data entered into the grid such as:

| CASE/VAR | FLU | GROUP |
|----------|-----|-------|
| CASE 1   | 0   | 1     |
| CASE 2   | 1   | 1     |
| I        |     |       |
| CASE 110 | 0   | 1     |
| CASE 111 | 0   | 2     |
|          |     |       |

## CASE 231 1 2

then the option would have been to analyze data in a file.

In this case, the absence or presence of flu symptoms for the student are entered as zero (0) or one (1) and the grade is coded as 1 or 2. If the same students, say the eighth grade students, are observed at weeks 10 and 15 during the semester, than the test assumptions would be changed to Dependent Proportions. In that case the form changes again to accommodate two variables coded zero and one to reflect the observations for each student at weeks 10 and 15.



Figure 2 Testing Equality of Two Independent Proportions (Grid Data)

### **Correlation Differences**

When two or more samples are obtained, the investigator may be interested in testing the hypothesis that the two correlations do not differ beyond that expected due to random sampling variation. This test may be performed by selecting the correlation differences procedure in the comparison sub-menu of the statistics menu. The following form then appears:

| Comparison of Correlations  | ×   |
|---|---|
| Data Entry By:<br>Values Entered On This Form<br>Values in the Data Grid    | Assume:<br><ul> <li>Independent Correlations</li> <li>Dependent Correlations</li> </ul> |
| First Correlation:.75Sample Size 1:30Second Correlation:.68Sample Size 2:40 |   |
|   |   |
|   |   |
| Percent Confidence Interval: 95   | Reset Cancel Continue   |

Figure 3 Test of Difference Between Two Independent Correlations

Notice that the form above permit the user to enter the correlations directly on the form or to compute the correlations for two groups by reading the data from the data grid. In addition, the form permits the user to test the difference between correlations where the correlations are dependent. This may arise when the same two variables are correlated on the same sample of subjects at two different time periods or on samples which are "matched" on one or more related variables. As an example, let us test the difference between a correlation of .75 obtained from a sample with 30 subjects and a correlation of .68 obtained on a sample of 40 subjects. We enter our values in the "edit" fields of the form and click the continue button. The results appear below:

COMPARISON OF TWO CORRELATIONS

```
Correlation one = 0.750
Sample size one = 30
Correlation two = 0.680
Sample size two = 40
Difference between correlations = 0.070
Confidence level selected = 95.0
z for Correlation One = 0.973
z for Correlation Two = 0.829
z difference = 0.144
Standard error of difference = 0.253
z test statistic = 0.568
Probability > |z| = 0.285
z Required for significance = 1.960
Note: above is a two-tailed test.
Confidence Limits = (-0.338, 0.565)
```

The above test reflects the use of Fisher's log transformation of a correlation coefficient to an approximate z score. The correlations in each sample are converted to z's and a test of the difference between the z scores is performed. In this case, the difference obtained had a relatively large chance of occurrence when the null hypothesis is true (0.285) and the 95% confidence limit brackets the sample difference of 0.253. The Fisher z transformation of a correlation coefficient is

$$z_r = \frac{1}{2} \log_e \left( \frac{1+r}{1-r} \right)$$

The test statistic for the difference between the two correlations is:

$$z_{r} = \frac{(z_{r_{1}} - z_{r_{2}}) - (z_{\rho_{1}} - z_{\rho_{2}})}{\sigma_{(z_{r_{1}} - z_{r_{2}})}}$$

where the denominator is the standard error of difference between two independent transformed correlations:

$$\sigma_{(z_{r_1}-z_{r_2})} = \sqrt{\left(\frac{1}{n_1-3}\right)\left(\frac{1}{n_2-3}\right)}$$

The confidence interval is constructed for the difference between the obtained z scores and the interval limits are then translated back to correlations. The confidence limit for the z scores is obtained as:

$$CI_{\%} = (z_{r_1} - z_{r_2}) + / - z_{\%}\sigma_{(z_{r_1} - z_{r_2})}$$

We can then translate the obtained upper and lower z values using:

$$r = \frac{e^{2z_r} - 1}{e^{2z_r} + 1}$$

For the test that two dependent correlations do not differ from zero we use the following t-test:

$$t = \frac{\left(r_{xy} - r_{xz}\right)\sqrt{(n-3)(1+r_{yz})}}{\sqrt{2\left(1-r_{xy}^{2} - r_{xz}^{2} - r_{yz}^{2} + 2r_{xy}r_{xz}r_{yz}\right)}}$$

### Tests for Two Means

### t-Tests

Among the comparison techniques the "Student" t-test is one of the most commonly employed. One may test hypotheses regarding the difference between population means for independent or dependent samples which meet or do not meet the assumptions of homogeneity of variance. To complete a t-test, select the t-test option from the Comparisons sub-menu of the Statistics menu. You will see the form below:

| Comparison of Two Sample Means                                |                  |                              |                              |          |  |  |  |
|---|------------------|------------------------------|------------------------------|----------|--|--|--|
| Data Entry By:<br>Values Entered On T<br>Values in the Data G | 'his Form<br>rid | Assume:<br>Indepe<br>Correla | endent Scores<br>ated Scores |          |  |  |  |
| Mean 1:   | Std. Dev. 1:     |                              | Sample Size 1:               |          |  |  |  |
| Mean 2:   | Std. Dev. 2:     |                              | Sample Size 2:               |          |  |  |  |
|   |                  |                              |                              |          |  |  |  |
|   |                  |                              |                              |          |  |  |  |
|   |                  |                              |                              |          |  |  |  |
|   |                  |                              |                              |          |  |  |  |
|   |                  |                              |                              |          |  |  |  |
|   |                  |                              |                              |          |  |  |  |
|   |                  |                              |                              |          |  |  |  |
|   | 95               | Revel 1                      | Grand                        |          |  |  |  |
| Percent Confidence Interv                                     | al: 50           | Heset                        | Cancel                       | Continue |  |  |  |

Figure 4 Dialog Form For The Student t-Test

Notice that you can enter values directly on the form or from a file read into the data grid. If you elect to read data from the data grid by clicking the button corresponding to "Values Computed from the Data Grid" you will see that the form is modified as shown below.





Figure 6 Student t-Test For Data in the Data Grid

We will analyze data stored in the Hinkle411.TEX file.

Once you have entered the variable name and the group code name you click the Continue button. The following results are obtained for the above analysis:

```
COMPARISON OF TWO MEANS
```

```
Variable
                      Variance Std.Dev.
                                         S.E.Mean N
              Mean
Group 1
             31.00
                       67.74
                                  8.23
                                           1.68 24
Group 2
             25.75
                       20.80
                                  4.56
                                           0.93 24
Assuming equal variances, t =
                                2.733 with probability = 0.0089 and
46 degrees of freedom
Difference =
                5.25 and Standard Error of difference =
                                                           1.92
Confidence interval = (
                          1.38,
                                 9.12)
Assuming unequal variances, t = 2.733 with probability = 0.0097 and
35.91 degrees of freedom
Difference = 5.25 and Standard Error of difference =
                                                           1.92
Confidence interval = (
                         1.35,
                                  9.15)
```

F test for equal variances = 3.256, Probability = 0.0032
NOTE: t-tests are two-tailed tests.

The F test for equal variances indicates it is reasonable to assume the sampled populations have unequal variances hence we would report the results of the test assuming unequal variances. Since the probability of the obtained statistic is rather small (0.01), we would likely infer that the samples were drawn from two different populations. Note that the confidence interval for the observed difference is reported.

What do you call a tea party with more than 30 people? A Z party!!!

## One, Two or Three Way Analyses of Variance

Analysis of Variance is one of the most commonly used methods for testing hypotheses of differences among means of samples collected from one or more populations. Typically there is a dependent variable and one to three "treatments" consisting of two or more "levels". To demonstrate this procedure, we will use a file labeled Anova2.LAZ. This file contains a dependent variable and three independent variables. The dependent variable X is a "floating point" type of variable. The three independent variables are row, column and slice and are coded as integer types of variables. We start our analysis by selecting this option and entering the variables to be analyzed. We will ignore the two "covariate" measures at this time.

| 🛞 One, Two or Three Way Analysi | s of ¥ariance  |   |   |                    |
|---------------------------------|--|---|---|--------------------|
| Variables:<br>Cov1<br>Cov2      | Dependent<br>X   | Directions: You may elect<br>ANOVA by selecting a dep<br>3 factor variables. If you e<br>comparisons are made bet<br>some post-hoc comparison<br>N's. | to complete a 1, 2 or 3 way<br>endent variable and 1, 2 or<br>elect post-hoc tests,<br>ween factor levels. NOTE:<br>s are made only with equal  |                    |
| Alpha Level for Overall Tests   | Factor 1 Variable Row Factor 2 Variable Col Factor 3 Clark Slice Alpha Level for Post-Hoc Tests: | Variable Type Factor 1 Factor 1 Factor 2 Factor 2 Factor 2 Factor 2 Factor 3 Random Levels 0.05 | Post-Hoc Comparisons: -<br>Scheffe<br>Tukey HSD (= n's)<br>Tukey B (= n's)<br>Tukey-Kramer<br>Newman-Keuls (= n's<br>Bonferroni<br>Orthogonal Contrasts<br>Options<br>Plot Means Using 3D<br>Plot Means Using 3D<br>Plot Means Using 3D | )<br>bars<br>Lines |
|                                 | Reset Cancel   | Compute   | Return  |                    |

Notice that each of the independent variables may be one of two types – fixed or random levels. We have also selected a "post-hoc" test as well as the option to plot means using three dimension bars. When we click the Compute button, we receive the output shown below.

Three Way Analysis of Variance

Variable analyzed: X

Factor A (rows) variable: Row (Fixed Levels) Factor B (columns) variable: Col (Fixed Levels) Factor C (slices) variable: Slice (Fixed Levels) SOURCE D.F. SS MS PROB.> F Omega Squared F 12.250 12.250 12.250 0.002 Among Rows 1 0.083 42.250 42.250 Among Columns 1 42.250 0.000 0.304

Among Slices 2 6.500 3.250 3.250 0.056 0.033 A x B Inter. 1 12.250 12.250 12.250 0.002 0.083 A x C Inter. 2 6.500 3.250 3.250 0.056 0.033 B x C Inter. 2 6.500 3.250 3.250 0.056 0.033 AxBxC Inter. 2 24.500 12.250 12.250 0.000 0.166 Within Groups 24 24.000 1.000 35 134.750 Total 3.850

Omega squared for combined effects = 0.735

Note: MSErr denominator for all F ratios.

**Descriptive Statistics** 

| GRO   | UP |   |   | Ν  | MEA   | N VAR | IANCE | STD.DEV. |
|-------|----|---|---|----|-------|-------|-------|----------|
| Cell  | 1  | 1 | 1 | 3  | 2.000 | 1.000 | 1.000 |          |
| Cell  | 1  | 1 | 2 | 3  | 3.000 | 1.000 | 1.000 |          |
| Cell  | 1  | 1 | 3 | 3  | 4.000 | 1.000 | 1.000 |          |
| Cell  | 1  | 2 | 1 | 3  | 5.000 | 1.000 | 1.000 |          |
| Cell  | 1  | 2 | 2 | 3  | 4.000 | 1.000 | 1.000 |          |
| Cell  | 1  | 2 | 3 | 3  | 3.000 | 1.000 | 1.000 |          |
| Cell  | 2  | 1 | 1 | 3  | 2.000 | 1.000 | 1.000 |          |
| Cell  | 2  | 1 | 2 | 3  | 5.000 | 1.000 | 1.000 |          |
| Cell  | 2  | 1 | 3 | 3  | 2.000 | 1.000 | 1.000 |          |
| Cell  | 2  | 2 | 1 | 3  | 5.000 | 1.000 | 1.000 |          |
| Cell  | 2  | 2 | 2 | 3  | 6.000 | 1.000 | 1.000 |          |
| Cell  | 2  | 2 | 3 | 3  | 8.000 | 1.000 | 1.000 |          |
| Row   | 1  | 1 |   | 18 | 3.500 | 1.676 | 1.295 |          |
| Row   | 2  | 2 |   | 18 | 4.667 | 5.529 | 2.351 |          |
| Col   | 1  |   |   | 18 | 3.000 | 2.118 | 1.455 |          |
| Col   | 2  |   |   | 18 | 5.167 | 3.324 | 1.823 |          |
| Slice | 1  |   |   | 12 | 3.500 | 3.182 | 1.784 |          |
| Slice | 2  |   |   | 12 | 4.500 | 2.091 | 1.446 |          |
| Slice | 3  |   |   | 12 | 4.250 | 6.386 | 2.527 |          |
| TOTA  | ٩L |   |   | 36 | 4.083 | 3.850 | 1.962 |          |

# TESTS FOR HOMOGENEITY OF VARIANCE

Hartley Fmax test statistic = 1.00 with deg.s freedom: 4 and 2. Cochran C statistic = 0.08 with deg.s freedom: 4 and 2. Bartlett Chi-square statistic = 0.00 with 3 D.F. Prob. larger = 1.000

COMPARISONS AMONG COLUMNS WITHIN EACH ROW

ROW 1 COMPARISONS

Scheffe contrasts among pairs of means. alpha selected = 0.05 Group vs Group Difference Scheffe Critical Significant? Statistic Value

1 2 1.00 1.22 2.093 NO

-----

ROW 2 COMPARISONS

\_\_\_\_\_

Scheffe contrasts among pairs of means. alpha selected = 0.05 Group vs Group Difference Scheffe Critical Significant? Statistic Value

1 2 -6.00 7.35 2.093 YES

#### COMPARISONS AMONG ROWS WITHIN EACH COLUMN

#### COLUMN 1 COMPARISONS

Scheffe contrasts among pairs of means. alpha selected = 0.05 Group vs Group Difference Scheffe Critical Significant? Statistic Value

1 2 2.00 2.45 2.093 YES

#### COLUMN 2 COMPARISONS

|      | S       | cheffe co<br>alpl | ntrasts a           | among patted = 0.05 | irs of mea | ns.          |
|------|---------|-------------------|---------------------|---------------------|------------|--------------|
| Grou | ıp vs G | roup Dif<br>Sta   | ference<br>tistic V | Scheffe<br>alue     | Critical   | Significant? |
| 1    | 2       | -5.00             | 6.12                | 2.093               | YES        |              |

-----



















## Within Subjects Analysis of Variance

Multiple independent treatments may be administered to the same subjects. This design offers the advantage of lower errors by not introducing between group errors. We will use the file labeled itemdata.LAZ to demonstrate. Test items administered to subjects are essentially independent measures of the knowledge the subjects have for a certain topic. As such the analysis of variance for these repeated measures also serves as a basis for estimating the test reliability. The theory of this method was initially developed by Hoyt. Here then is the form that appears to complete the analysis. Note the options we have selected.

| 😵 Within Subje                                      | cts ANOVA and Hoyt R   | eliability Estimates  |   |       |  |  |
|---|--|---|---|-------|--|--|
|   | Directions: The repeated measures ANOVA requires you to select two or<br>more<br>variables (columns) which represent repeated observations on the same<br>subjects (rows.) Homogeneity of variance and covariance are assumed and<br>may be tested as an option. In addition, the ANOVA provides the basis for<br>estimates of reliability as developed by Hoyt (Intraclass reliability) with the<br>adjusted estimate equivalent to the Cronbach Alpha estimate. Finally, you |   |   |       |  |  |
| Available Variable<br>LastName<br>FirstName<br>IDNO | 25   | Selected Variables:<br>VAR1<br>VAR2<br>VAR3<br>VAR4<br>VAR5 | Options:<br>Reliability Estimates<br>Plot Means<br>Reset<br>Compute | ancel |  |  |

When we press the Compute button we obtain:

Treatments by Subjects (AxS) ANOVA Results.

Data File = C:\lazarus\Projects\LazStats\itemdat.LAZ

| SOURCE   | DF                         | SS                                 | MS                                    | F Prob.        | > F   |
|--|----------------------------|------------------------------------|---------------------------------------|----------------|-------|
| SUBJECTS<br>WITHIN SUB<br>TREATMEN<br>RESIDUAL | 15<br>JECTS<br>NTS 4<br>60 | 6.350<br>64 13<br>4 3.05<br>10.150 | 0.423<br>0.200 0.<br>0 0.763<br>0.169 | 206<br>3 4.507 | 0.003 |
| TOTAL  | 79 1                       | 9.550                              | 0.247                                 |                |       |

\_\_\_\_\_

 TREATMENT (COLUMN) MEANS AND STANDARD DEVIATIONS

 VARIABLE MEAN
 STD.DEV.

 VAR1
 0.875
 0.342

 VAR2
 0.688
 0.479

 VAR3
 0.563
 0.512

 VAR4
 0.438
 0.512

 VAR5
 0.313
 0.479

Mean of all scores = 0.575 with standard deviation = 0.497

RELIABILITY ESTIMATES

TYPE OF ESTIMATEVALUEUnadjusted total reliability0.513Unadjusted item reliability0.174Adjusted total (Cronbach)0.600Adjusted item reliability0.231

#### BOX TEST FOR HOMOGENEITY OF VARIANCE-COVARIANCE MATRIX

#### SAMPLE COVARIANCE MATRIX with 16 cases.

Variables

|      | VAR1   | VAR2  | VAR3  | VAR4   | VAR5  |
|------|--------|-------|-------|--------|-------|
| VAR1 | 0.117  | 0.025 | 0.008 | -0.008 | 0.042 |
| VAR2 | 0.025  | 0.229 | 0.121 | 0.079  | 0.037 |
| VAR3 | 0.008  | 0.121 | 0.263 | 0.071  | 0.079 |
| VAR4 | -0.008 | 0.079 | 0.071 | 0.263  | 0.054 |
| VAR5 | 0.042  | 0.037 | 0.079 | 0.054  | 0.229 |

#### ASSUMED POP. COVARIANCE MATRIX with 16 cases.

Variables

|      | VAR1  | VAR2  | VAR3  | VAR4  | VAR5  |
|------|-------|-------|-------|-------|-------|
| VAR1 | 0.220 | 0.011 | 0.011 | 0.011 | 0.011 |
| VAR2 | 0.048 | 0.219 | 0.010 | 0.010 | 0.010 |
| VAR3 | 0.048 | 0.046 | 0.219 | 0.010 | 0.010 |
| VAR4 | 0.048 | 0.046 | 0.044 | 0.219 | 0.009 |
| VAR5 | 0.048 | 0.046 | 0.044 | 0.042 | 0.218 |

Determinant of variance-covariance matrix = 0Determinant of homogeneity matrix = 0ChiSquare = 56.769 with 13 degrees of freedom Probability of larger chisquare = 6.96E-007



# The A by S Analysis of Variance

One can apply repeated measures to subjects in two or more separate groups. For example, we may be interested in the differences between males and females sampled from a school that have been administered a standardized achievement test. We will use the ABRDATA.LAZ file to demonstrate this analysis. Notice the variables we have selected and the options chosen:



When we click the Compute button, the following results are observed:

ANOVA With One Between Subjects and One Within Subjects Treatments

| Source                | df      | SS                | MS         | F       | Prob.  |
|-----------------------|---------|-------------------|------------|---------|--------|
| Between<br>Groups (A) | 11<br>1 | 181.000<br>10.083 | 10.083     | 0.590   | 0.4602 |
| Subjects w.g          | . 10    | 1/0.91/           | 17.092     |         |        |
| Within Subject        | ts 36   | 5 1077.00         | 0          |         |        |
| B Treatments          | s 3     | 991.500           | 330.500    | 128.627 | 0.0000 |
| A X B inter.          | 3       | 8.417             | 2.806      | 1.092   | 0.3677 |
| B X S w.g.            | 30      | 77.083            | 2.569      |         |        |
| TOTAL                 | 47      | 1258.000          |            |         |        |
| Means                 |         |                   |            |         |        |
| TRT. B 1 B            | 2 B     | 3 B 4             | TOTAL      |         |        |
| А                     |         |                   |            |         |        |
| 1 16 167 11           | 000 7   | 7 833 3 16        | 7 9 542    |         |        |
| 2 16 833 12           | 000 7   | 1667 5 33         | 3 10 458   |         |        |
| TOTAL 16 50           | 0115    | 00 7 750          | 4 250 10 0 | 00      |        |
| 101AL 10.30           | 0 11.5  | 00 7.750          | 4.230 10.0 | 00      |        |

Standard Deviations TRT. B 1 B 2 B 3 B 4 TOTAL A 1 2.714 2.098 2.714 1.835 5.316 2 1.329 2.828 2.338 3.445 5.099 TOTAL 2.067 2.431 2.417 2.864 5.174



## The A x B x R Analysis of Variance

Like the previous "mixed" design ANOVA that had repeated measures for different levels of treatment for one independent variable, we can also combine a two-way ANOVA with repeated measures. We will demonstrate using the same data file as in the previous analysis, namely the ABRDATA.LAZ file.

| 😤 AxBxR ANOVA (two between and repeated measures)   | _ 🗆 ×                      |
|---|----------------------------|
| The AxBxR ANOVA involves two between treatment factors and repeated measures factors for two grid column variables contain the A and B treatment values (codes 1, 2, etc.) and 2 to K grid column variables for the repeated measure observations. All ABC groups are assumed to be of the same size. There is a maximum of 20 repeated measures. | rrs.<br>D                  |
| Available Variables:<br>Factor A Variable<br>Row<br>Factor B Variable<br>Col<br>Repeated Measures<br>C1<br>C2<br>C3<br>C4   | Reset<br>Cancel<br>Compute |
| Plot Means     Test Homogeneity of Covariance   | Return                     |

When you click the Compute button you obtain:

| SOURCE           | DF   | SS       | MS      | F P     | ROB.  |
|------------------|------|----------|---------|---------|-------|
| Between Subjects | 11   | 181.000  |         |         |       |
| A Effects        | 1    | 10.083   | 10.083  | 0.978   | 0.352 |
| B Effects        | 1    | 8.333    | 8.333   | 0.808   | 0.395 |
| AB Effects       | 1    | 80.083   | 80.083  | 7.766   | 0.024 |
| Error Between    | 8    | 82.500   | 10.313  |         |       |
| Within Subjects  | 36   | 1077 000 |         |         |       |
| C Replications   | 3    | 991 500  | 330 500 | 152 051 | 0 000 |
| AC Effects       | 3    | 8.417    | 2.806   | 1.291   | 0.300 |
| BC Effects       | 3    | 12.167   | 4.056   | 1.866   | 0.162 |
| ABC Effects      | 3    | 12.750   | 4.250   | 1.955   | 0.148 |
| Error Within     | 24   | 52.167   | 2.174   |         |       |
| Total            | 47   | 258.000  |         |         |       |
| ABR Means Table  | with | 3 cases. |         |         |       |
| Variables        |      |          |         |         |       |

|       | C1     | C2     | C3    | C4    |
|-------|--------|--------|-------|-------|
| A1 B1 | 17.000 | 12.000 | 8.667 | 4.000 |

| A1 B2     | 15 333     | 10 000      | 7 000 | 2 333   |
|-----------|------------|-------------|-------|---------|
| A2 B1     | 16 667     | 10,000      | 6.000 | 2 333   |
| 112 D1    | 10.007     | 10.000      | 0.000 | 2.555   |
| A2 B2     | 17.000     | 14.000      | 9.333 | 8.333   |
|           |            |             |       |         |
| AB Means  | Table with | h 12 cases. |       |         |
| Variables |            |             |       |         |
|           | B 1        | В2          |       |         |
| A1        | 10.417     | 8.667       |       |         |
| 12        | 0 750      | 12 167      |       |         |
| AZ        | 8.730      | 12.107      |       |         |
|           |            |             |       |         |
| AC Means  | Table with | h 6 cases.  |       |         |
| Variables |            |             |       |         |
|           | C 0        | C 1         | C 2   | C 3     |
| A0        | 16.167     | 11.000      | 7.833 | 3.167   |
| A 1       | 16 833     | 12 000      | 7 667 | 5 333   |
| 111       | 10.055     | 12.000      | 1.007 | 5.555   |
| -         |            |             |       |         |
| BC Means  | Table with | h 6 cases.  |       |         |
| Variables |            |             |       |         |
|           | C 1        | C 2         | C 3   | C 4     |
| B1        | 16.833     | 11.000      | 7.333 | 3.167   |
| ЪJ        | 16 167     | 12 000      | 8 167 | 5 3 3 3 |
| DL        | 10.107     | 12.000      | 0.10/ | 5.555   |

Variance-Covariance AMatrix for A1 B1 with 12 cases. Variables

|    | C1    | C2     | C3     | C4    |
|----|-------|--------|--------|-------|
| C1 | 7.000 | 9.500  | 8.667  | 5.583 |
| C2 | 4.000 | 10.000 | 10.083 | 6.667 |
| C3 | 5.000 | 11.000 | 12.333 | 7.167 |
| C4 | 4.000 | 11.000 | 10.667 | 8.167 |

Variance-Covariance AMatrix for A1 B2 with 12 cases. Variables

|    | C1     | C2    | C3     | C4    |
|----|--------|-------|--------|-------|
| C1 | 9.333  | 8.750 | 4.333  | 2.125 |
| C2 | 4.000  | 9.000 | 9.042  | 5.333 |
| C3 | 0.000  | 9.500 | 13.167 | 7.583 |
| C4 | -0.667 | 7.500 | 9.333  | 6.417 |

Variance-Covariance AMatrix for A2 B1 with 12 cases. Variables

|    | C1     | C2    | C3     | C4     |
|----|--------|-------|--------|--------|
| C1 | 1.333  | 2.375 | 0.167  | -0.271 |
| C2 | -2.000 | 8.500 | 6.521  | 3.667  |
| C3 | -2.000 | 6.750 | 10.583 | 6.792  |
| C4 | -1.333 | 4.750 | 7.667  | 5.542  |

Variance-Covariance AMatrix for A2 B2 with 12 cases. Variables

|    | C1     | C2    | C3    | C4     |
|----|--------|-------|-------|--------|
| C1 | 3.000  | 4.188 | 1.083 | -0.635 |
| C2 | 3.000  | 8.250 | 5.260 | 1.833  |
| C3 | 1.000  | 5.375 | 6.625 | 3.729  |
| C4 | -0.500 | 2.375 | 4.167 | 3.104  |

Pooled Variance-Covariance AMatrix with 12 cases. Variables

|    | C1    | C2    | C3     | C4    |
|----|-------|-------|--------|-------|
| C1 | 5.167 | 6.203 | 3.563  | 1.701 |
| C2 | 2.250 | 8.938 | 7.727  | 4.375 |
| C3 | 1.000 | 8.156 | 10.677 | 6.318 |
| C4 | 0.375 | 6.406 | 7.958  | 5.807 |

Test that sample covariances are from same population: Chi-Squared := 11.222 with 30 degrees of freedom. Probability of > Chi-Squared := 0.999

Test that variance-covariances AMatrix has equal variances and equal covariances: Chi-Squared := 8.589 with 8 degrees of freedom. Probability of > Chi-Squared := 0.378







## Analysis of Covariance by Multiple Regression

All of the analysis of variance designs may be considered as different problems in multiple regression. The model of each ANOVA is actually a multiple regression model. In some cases, it is easier to specify the analysis as a multiple regression equation to do the analysis than to "partition" variance into separate components as is done for many of the more simple designs. This procedure demonstrates the use of multiple regression to obtain an analysis of covariance. We will use the file labeled ANCOVA.LAZ. When you choose this analysis option, you see the form below:

| 😵 Analysis of Covariance Using Multi | ple Regression Methods |  |
|--------------------------------------|------------------------|--|
| Available Variables:                 | Dependent Variable     | This procedure analyzes fixed effects with up to three levels of<br>interaction and one or more covariates. Multiple regression<br>methods<br>are used (See "Multiple Regression in Behavioral Research" by<br>Elazar J. Pedhazur, Harcourt, Brace, College Publishers, 1997,<br>Chapter 16, pages 675-713.) A test is performed for the<br>assumption of homogeneous regression slopes in addition to the |
|                                      | Group                  | Output Options:     Reset       Image: Descriptive Statistics     Reset       Image: Correlation Matrices     Cancel       Image: Inverse of Matrices     Cancel   |
|                                      | Covariates<br>X<br>Z   | Plot Factor Means     Compute     Show Multiple Comparisons     Return   |
|                                      |                        |  |

Clicking the compute button yields the results displayed next. Examine your grid data following the output results. You will see that additional variables have been created that reflect the contributions of each level of each treatment variable using effect coding.





#### ANALYSIS OF COVARIANCE USING MULTIPLE REGRESSION

File Analyzed: C:\lazarus\Projects\LazStats\ANCOVA.LAZ

Model for Testing Assumption of Zero Interactions with Covariates

MEANS with 40 valid cases.

Variables

Х

7.125 14.675

Ζ

A1 0.000

Variables Х Ζ A1 A2 A3 7.125 0.000 0.000 0.000 14.675 Variables XxA1 XxA2 XxA3 ZxA1 ZxA2 0.125 0.025 0.075 -0.400 -0.125 Variables ZxA3 Υ -0.200 17.550 VARIANCES with 40 valid cases. Variables Х Ζ A1 A2 A3 4.163 13.866 0.513 0.513 0.513 Variables ZxA1 ZxA2 XxA1 XxA2 XxA3 28.010 27.712 116.759 27.102 125.035 Variables ZxA3 Y 124.113 8.254 STD. DEV.S with 40 valid cases. Variables Х Ζ A1 A2 A3 2.040 3.724 0.716 0.716 0.716 Variables XxA1 XxA2 XxA3 ZxA1 ZxA2 5.292 5.206 5.264 10.806 11.182 Y Variables ZxA3 11.141 2.873 Analysis of Variance for the Model to Test Regression Homogeneity Prob>F SOURCE Deg.F. MS SS F Explained 11 228.08 20.73 6.188 0.0000 93.82 Error 28 3.35 39 321.90 Total R Squared = 0.709 Model for Analysis of Covariance MEANS with 40 valid cases.

A2

0.000

A3

0.000

Variables Y 17.550 VARIANCES with 40 valid cases. Ζ Variables Х A1 A2 A3 4.163 13.866 0.513 0.513 0.513 Variables Y 8.254 STD. DEV.S with 40 valid cases. Variables Х Ζ A1 A2 A3 2.040 3.724 0.716 0.716 0.716 Variables Y 2.873 Test for Homogeneity of Group Regression Coefficients Change in R2 = 0.0192. F = -0.308 Prob.> F = 0.9275 with d.f. 6 and R Squared = 0.689 Analysis of Variance for the ANCOVA Model SOURCE Deg.F. SS MS F Prob>F Explained 5 221.89 44.38 15.087 0.0000 Error 34 100.01 2.94 39 321.90 Total Unadjusted Group Means for Group Variables Group Means with 40 valid cases. Variables 15.800 17.900 19.100 17.400 Intercepts for Each Group Regression Equation for Variable: Group Inercepts with 40 valid cases. Group 2 Variables Group 1 Group 3 Group 4 8.076 10.505 11.528 10.076 Adjusted Group Means for Group Variables Group Means with 40 valid cases. Variables Group 1 Group 2 Group 3 Group 4 15.580 18.008 19.032 17.579

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| SOURCE | Deg | g.F. SS | MS    | F      | Prob>F |
|--------|-----|---------|-------|--------|--------|
| Cov0   | 1   | 78.70   | 78.70 | 26.754 | 0.0000 |
| Cov1   | 1   | 0.66    | 0.66  | 0.225  | 0.6379 |
| А      | 3   | 60.98   | 20.33 | 6.911  | 0.0009 |
| ERROR  | 34  | 100.01  | 2.94  |        |        |

Test for Each Source of Variance - Type III SS